Firm-Dynamics under Imperfect Information: Selection and the Relative Firm Size Distribution

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Abstract

I study the impact of entry-stage information frictions on consumption-equivalent welfare in the US macroeconomy. The framework for this exercise is a simple Chamberlinian model with heterogenous firms, entry-stage selection, and rigidities in capital adjustments. The severity of information frictions is governed by the precision of a private signal on firm-level fundamentals. Leveraging the information content of both *exit rates* and *capital adjustments* among comparatively young establishments, the model is calibrated to US Census of Manufactures and BDS data. A diminution of information frictions over time is found to be consistent with a number of well-documented secular trends: a rise in concentration, an increase in profitability, a decoupling of wage- and productivity-growth. The welfare-gains to be had from a hypothetical elimination of the entirety of entry-stage information frictions are in the ballpark of roughly 10%.

Keywords: Entry, Information Frictions, Welfare, Concentration.

JEL Codes: D4, D8, L1.

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1 Introduction

This paper explores the response of macro-aggregates to a diminution of entry-stage information frictions in the US economy. Particular emphasis is placed on the study of welfare-implications. Underscoring the relevance of this exercise, I provide reduced-form evidence on an improvement of private-sector forecasting practices over the past 15 years. Building on a simple structural model of firm dynamics under imperfect information, I then establish that a decline of information frictions is, by and large, consistent with a number of salient features of the data.

Over the past decades the U.S. economy has witnessed the unfolding of a number of well-documented secular trends: a rise in concentration, an increase in profitability, a decline of the labor share, a decoupling of wage- and productivity-growth, as well as an overall decrease in business dynamism. Aimed at the explanation of these trends, there is a vast body of scholarship on the state of competition documenting changes in the economic environment that promote an evermore stringent exertion of market power. Dottling, Guttierez, and Philippon (2018), for instance, argue that a weakening of U.S. antitrust enforcement finds itself reflected in a diminution of competitive pressure. Offering an alternative perspective, Autor, Dorn, Katz, Patterson, and van Reenen (2021) advert to the fact that globalization entails a rise in the toughness of competition. Production is, thus, reallocated towards the most productive firms driving up markups and depressing the labor share. Matching time-series on markups, labor market dynamism, and overhead, De Loecker, Eeckhout, and Mongey (2021) find that changes in market power are driven by the distinct channels of technology and market structure.

With the typical narrative revolving around rent-seeking behaviour, the literature has paid little attention to the potential refinement of selection resulting from the concurrent advancements in information technology. There is little doubt that information is much more readily available and more efficiently processed today than it has been 20 years ago. It would, therefore, seem natural to conjecture that establishments on the verge of entry do, in fact, exploit this wealth of information to forecast uncertain fundamentals. Even though clearly not the sole driver of the aforementioned trends, the adoption of evermore sophisticated forecasting protocol does, in fact, turn out to be consistent with the time-series evolution of several key macro-aggregates in the data.

In particular, I study the impact of information frictions through the lens of a (2003) Melitz-type model where endogenous entry furnishes the economy with a selection channel for information to affect the relative firm size distribution. An endogenous measure of heterogeneous potential entrants observe a private signal on their idiosyncratic productivity. Contingent on entry, i.e. the incurrence of an entry cost, establishments learn of their respective fundamental at which point they either take up production or alternatively decide to leave the market. Selection, therefore, plays out in two stages: an entry decision governed by a signal-cutoff as well as a production decision governed by a productivity-cutoff. The degree of information frictions in the economy is captured in terms of the parameter index on signal precision. In order to leverage the information content of capital adjustments following the imperfectly informed choice of an *initial* capital stock, the model-framework is augmented to feature a notion of rigidities in the employment of capital.

Organization. The remainder of this paper is structured as follows: Section 2 offers some reduced-form evidence for a diminution of information frictions. Section 3 delineates a structural model aimed at an assessment of the welfare-implications of a decline in the severity of information frictions. Section 4 is the dedicated to the model's calibration as well as the discussion of a few select predictions. Section 5 summarizes.

2 Motivating evidence

There is certainly a compelling case to be made that information is much more readily available today than, say, two decades ago. We have an unprecedented environment for data collection, academia has made significant headway in developing forecasting theory, and – perhaps most crucially – we have the computational power to implement extraordinarily sophisticated prediction algorithms. The subsequent section provides some reduced-form evidence that firms do, in fact, exploit these advancements in information technology.

Data and measurement. I look at annual 10-K reports filed with the Securities and Exchange Commission (SEC) in order to ascertain whether the evolution of language patterns therein suggests the adoption of evermore sophisticated forecasting protocol. Since 10-K filings are readily accessible to (potential) investors, the private sector has an incentive to use these filings in order to communicate forecasting expertise. Barring a few natural caveats to be addressed shortly, it follows that the choice of *natural language* in these filings should paint a somewhat accurate picture of both the extent as well as the sophistication of data-usage in informing management decisions. The data source for this exercise is the full text of annual 10-K reports of

US publicly traded firms submitted to the SEC during the time-period from 2006 to 2021. Due to their inherent scope for forward-looking statements, the main-analysis is confined to items 1A, 7, and 7A.[1] The relevant digitized documents are hosted on the SEC EDGAR database which began operation in 1994. The choice of 2006 as a starting point for the exercise at hand allows for a consistent extraction of the pertinent items vis-a-vis filing conventions.

Methodology. The main-analysis builds on a heuristic in computational linguistics which has become a standard in economic research that concerns itself with text classification. Specifically, the following exercise is predicated on a *term-frequency inverse-document-frequency* (tf-idf) classification protocol. By way of example, Flynn and Sastry (2022) offer a state-of-the-art tf-idf analysis of macro-attentiveness as conveyed via the choice of language in 10-K and 10-Q filings. Generally speaking, the idea is to ascertain to what extent a particular term is characteristic of a particular filing measured against a fixed document corpus. Regarding the evolution of private sector forecasting practices, the question becomes whether forecast-specific language that is characteristic of *more recent* 10-K filings is reflective of the adoption of *more sophisticated* forecasting methodology.

In order to identify language that is specific to forecasting in a manner that is broadly free of researcher bias, I follow Hassan et al. (2019) and construct a set of reference libraries. These libraries are, respectively, based on introductory, intermediate, as well as advanced textbooks on econometrics, statistics, and forecasting. My choice of reference texts deliberately encompasses a wide array of vocabulary ranging from the non-technical (e.g. prediction, estimate) to the highly specialized (e.g. heteroskedasticity, attenuation bias). The intended readership of the textbooks immediately lends itself to a classification of their respective sophistication. I cover an extensive part of the literature in order to weed out language idiosyncrasies, author bias, and patterns due to concept origination and diffusion; the latter is separately addressed in a robustness check.

Let $(\mathscr{B}, \succeq)$ denote a linearly ordered version of the universe of English language bigrams and take $\mathscr{T}$ to be the sample of textbooks used in this exercise. Upon removal of stop-words, a textbook $T \in \mathscr{T}$ is reduced to a vector of $\succeq$-ordered counts of bigrams such that $T \in \mathbb{N}^{|\mathscr{B}|}$. For any bigram $b \in \mathscr{B}$ its term-frequency in textbook T is prescribed as

$$\mathtt{tf}_T(b) = \langle T, \mathbf{1}_b \rangle$$

[1]Items 1A, 7, and 7A are concerned with *risk factors*, the *management's discussion and analysis of financial condition and results of operations*, and *quantitative and qualitative disclosures about market risk*, respectively.

where $\mathbf{1}_b$ is a selection vector for b in $(\mathscr{B}, \succeq)$. The inverse document-frequency in reference to a document corpus $\mathscr{R} = \mathscr{T} \cup \{\text{broader economic communications}\}$ is given as

$$\texttt{idf}_{\mathscr{R}}(b) = \log\left(\frac{|\mathscr{R}|}{1 + \sum_{\mathscr{R}} \mathbb{1}\{\langle R, \mathbf{1}_b \rangle \neq 0\}}\right) + 1.$$

Interacting term-frequency and inverse document-frequency to the effect that

$$\texttt{tf-idf}_T(b) = \texttt{tf}_T(b) \cdot \texttt{idf}_{\mathscr{R}}(b)$$

furnishes us with a measure of the extent to which a bigram $b \in \mathscr{B}$ is characteristic of a particular textbook $T \in \mathscr{T}$ measured against the fixed document corpus $\mathscr{R}$. Partitioning $\mathscr{T}$ along the dimension of isophistication, it follows that for each $s \in \{\text{introductory, intermediate, advanced}\}$

$$\texttt{score}_s(b) = \sum_{\mathscr{T}_s} \texttt{tf-idf}_T(b)$$

gives us a sense of the extent to which bigram b is characteristic of introductory, intermediate, and advanced texts on forecasting, respectively. Ranking bigrams accordingly, I recover sophistication-specific reference libraries by collecting the top 50 bigrams vis-a-vis $\mathscr{T}_{\text{introductory}}$. Then, upon exclusion the resulting library $\mathscr{L}_{\text{introductory}}$, I move on to an analogous construction of $\mathscr{L}_{\text{intermediate}}$ and, finally, $\mathscr{L}_{\text{advanced}}$.

Turning to the main-exercise, the relevant document corpus is now most conveniently described in terms of a mapping

$$D : \{\texttt{A}, \texttt{AA}, \texttt{AAACU}, \ldots, \texttt{ZZGQ}\} \times \{2006, 2007, \ldots, 2021\} \to \mathbb{N}^{|\mathscr{B}|}$$

such that a given 10-K filing is represented as a vector of bigrams ordered according to $(\mathscr{B}, \succeq)$ and indexed at both its stock ticker as well as filing year. Following the same line of reasoning as in the previous paragraph, a sophistication-specific measure of private-sector forecast attentiveness a time t is then recovered as

$$F_t^s = \sum_i \lambda_i \sum_{b \in \mathscr{L}_s} \texttt{td-idf}_{i,t}(b) \tag{1}$$

where λ_i allows for the adoption of different weighting schemes, e.g. in terms of relative market capitalization. By way of example, a comparatively (in a temporal sense) large value of, say, $F_{2019}^{\text{adv.}}$ points to the fact that the usage of forecast-specific vocabulary has grown more sophisticated in 2019. Provided that $F_{2019}^{\text{intr.}}$ is also large this is indicative of an overall increase in forecast attentiveness, otherwise one could deduce a mere adoption of more sophisticated forecasting practices.

An improvement in private sector forecast practices. In terms of results, when looking at 10-K filings, the occurrence of rather sophisticated terms – terms like *confidence interval, vector-autoregression,* and *Monte-Carlo simulation* – is much more commonplace today than it has been 15 years ago. Figure 1 illustrates this insight through a time-series for the forecast-attention index constructed in Equation (1) for $s \in \{\text{introductory}, \text{advanced}\}$.

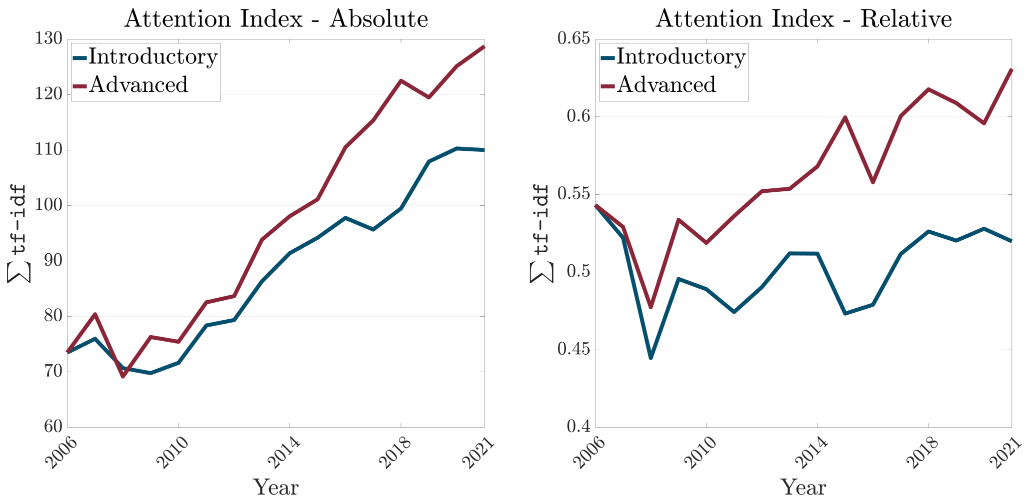

FIGURE 1. The time-series depicted above are constructed in accordance with (1). The upward directionality in the lefthandside panel is reflective of an increase in the usage of forecast-specific vocabulary over time. This increase is, however, more pronounced when confining analysis to comparatively sophisticated terminology. One might reasonably deduce an adoption of more and more advanced forecasting methodology. Predicated on a relative notion of term-frequency, the righthandside panel suggests that forecast attentiveness, itself, has stagnated over the past decade. Forecast sophistication did, however, also increase from this more conservative perspective.

The lefthandside panel in Figure 1 is generated in strict adherence to the standards set forth in papers that have been pioneering natural language processing in the realm of economic scholarship. The righthandside panel, on the other hand, is to be understood as something of a robustness check. In generating this graph, every degree of freedom is met with the most conservative judgement call possible.

Cheap talk concerns. There is certainly a compelling case to be made that executives might have just learned that touting the employment of *artificial intelligence, machine learning,* and *big data* instills a sense of false confidence in investors. In that case, the graphs in Figure 1 would merely document trends in the usage of vocabulary that is not actually backed by any substance whatsoever. Addressing these cheap talk concerns, the most striking result is that, controlling for size, the average

EBIT of a firm with $F_i > \bar{F}$ is roughly threefold the average EBIT of a firm with $F_i < \bar{F}$. The results of a pooled regression of earnings on $F_{i,t}$ are reported in Table 1.

Regressor	Estimate	Standard Error	t-Statistic	p-Value
Intercept	-25.364	39.384	-0.6440	0.5196
Size	0.0698***	0.0007	102.35	1.2989e-17
F	1.2955***	0.2871	4.5121	6.5584e-06

TABLE 1. Results of a – somewhat crude – pooled regression of company earnings (Compustat data on EBIT) on an averaged version of $F^{\text{adv.}}$ controlling for size. This is merely to establish correlation w/o taking into account unobserved heterogeneity of any sort.

Interestingly, $F_{i,t}$ is also significant against the regressand of earnings growth. By and large, it would seem that firms that communicate a higher degree of expertise in forecasting tend to outperform firms that do not.

A qualification. This project – and especially the structural model in the following section – concerns entry-stage information frictions. It would, therefore, seem reasonable to ask whether trends in the adoption of more sophisticated forecasting practices among entrenched (publicly traded) firms actually do extend to establishments at the verge of entry. In order to assuage this concern, it turns out that the patterns documented in Figure 1 come out even more pronounced when confining oneself to the analysis of S-1 filings – which are registration statements filed by companies preparing for IPO. A second mitigating factor is that, when thinking about establishment entry, a not insubstantial fraction of establishments are backed by the market research machinery of a large umbrella corporation. I do, however, acknowledge that there is a data availability issue.

Keeping in mind the trends documented in Figure 1, it would seem that it is – at the very least – a worthwhile endeavor to study the welfare implication of a further diminution of information frictions. The subsequent section delineates a simple structural model aimed at this objective.

3 A model with entry-stage information frictions

I study selection through the lens of a simple Chamberlinian model with entry-stage information frictions and rigidities in capital adjustments. Selection plays out through a comparatively layered channel of endogenous entry where entry decisions are, in particular, informed by a private signal on a firm-level fundamental.

3.1 The environment

Time is discrete and indexed at $t \in \mathbb{N}$. A single composite consumption good is marketed by a perfectly competitive final goods firm and produced through aggregation of an endogenous measure of imperfectly substitutable varieties. These varieties are supplied by a continuum of monopolistically competitive intermediate goods firms. Production of varieties takes place according to a CRS technology utilizing labour and capital as its only inputs. The is no aggregate uncertainty and analysis is confined to a stationary competitive equilibrium.

A representative household. With firm dynamics taking front and center stage in this paper, the household side is deliberately kept simple. There is a representative household that consumes final output, owns all firms, and furnishes the economy with an inelastic labor supply $L \in \mathbb{R}_+$. The latter is most instructively thought of as the size of the economy.

Intermediate goods firms. An endogenous measure N of intermediate goods firms i operate a Cobb-Douglas production technology

$$y_i = \exp(z_i) n_i^{1-\alpha} k_i^{\alpha}. \tag{2}$$

Conditional on labor inputs n_i and capital inputs k_i the only dimension of heterogeneity is encapsulated in terms of a firm-level fundamental z_i.[2] In the sense of an accounting exercise, firm-level profits π_i are pinned down as revenue net of firm i's wage bill, capital cost, as well as a fixed cost of operation ϕ that is denominated in units of final output

$$\pi_i = p_i y_i - W n_i - R k_i - \phi.$$

The economy's numeraire is final output with its price normalized to unity.

The final goods firm. The demand structure in the market for varieties arises from profit maximization of a perfectly competitive final goods firm operating a CES aggregation technology. Aggregate output Y is, therefore, implicitly defined by

$$Y^{\frac{\sigma-1}{\sigma}} = \int_N y_i^{\frac{\sigma-1}{\sigma}} di$$

where $\sigma > 1$ denotes that elasticity of substitution between any given pair of varieties. Optimization dictates that varieties are purchased according to an iso-elastic

[2]Note that even though admitting an immediate interpretation as firm-level TFPQ, z_i might also be reflective of idiosyncratic demand shifts / product quality. For conciseness, it is, henceforth, referred to as either a firm-level fundamental or alternatively productive efficiency.

demand function, in the sense, that intermediate good firms face

$$y_i = Y p_i^{-\sigma}. \tag{3}$$

Capital cost. I adopt a small open economy setting to the effect that the user cost of capital are exogenous. That is to say,

$$R = \delta + r$$

where δ is to denote depreciation and r is to unresponsive to the aggregate employment of capital in production.

3.2 Firm dynamics

The sequence of decisions. Each period there is an unbounded measure of profit-seeking entities that contemplate the introduction of a novel variety. Upon payment of an information cost $\rho > 0$[3], these *would-be* potential entrants are free to run a market survey. The resultant sense of market transparency is modeled in terms of a private signal on their respective firm-level fundamental z_i

$$s_i = z_i + \tau \xi_i \tag{4}$$

where $z_i \sim H$ for some $H \in \Delta Z$ $(Z \subseteq \mathbb{R})$ while $\xi_i \sim G$ with $G \in \Delta\mathbb{R}$ such that $\mathbb{E}[\xi_i] = 0$, $\mathbb{E}[\xi_i^2] = 1$, and z_i independent of ξ_i. Crucially, the noise component in (4) is scaled at a parameter $\tau > 0$ that effectively governs the severity of information frictions. Since information is costly, equilibrium dictates that each period there is a finite mass of *actual* potential entrants that go ahead with this market survey. Based on their respective signal realization, potential entrants decide whether to enter (at an entry cost κ) where, if so, they are required to commit to an initial capital stock k.

Following Melitz (2003), upon entry, firms become cognizant of their time-invariant fundamental z_i. Crucially, the resolution of uncertainty takes place after capital commitments have already been struck up. Entrants then either take up operations or exit of their own volition. This margin of *early exit* is the only source of endogenous exit in the model. If operating, firms face an exogenous sequence of iid exit shocks that are entirely orthogonal to fundamentals. Parameterized at a Bernoulli rate φ, these exit shocks provide a mathematically convenient way to define the economy's stationary equilibrium.

[3] The information cost ρ is denominated in units of final output.

More interestingly, firms also face an exogenous sequence of capital adjustment shocks. Since the initial capital commitment is informed by a noisy measurement of z_i, learning of one's actual fundamental realization almost surely warrants an adjustment of capital inputs. The modeling assumption is that firms are not necessarily immediately free to act accordingly; there are rigidities to capital adjustments. In order to capture these rigidities in a parsimonious and tractable fashion I espouse a Calvo-type tradition. Each period a particular firm is free to adjust its capital inputs to their optimal level with a constant probability $1-\theta$. Since productive efficiency is time-invariant, once a firm receives its capital-adjustment shock, it subsequently produces at the optimal factor composition until it exits for exogenous reasons.

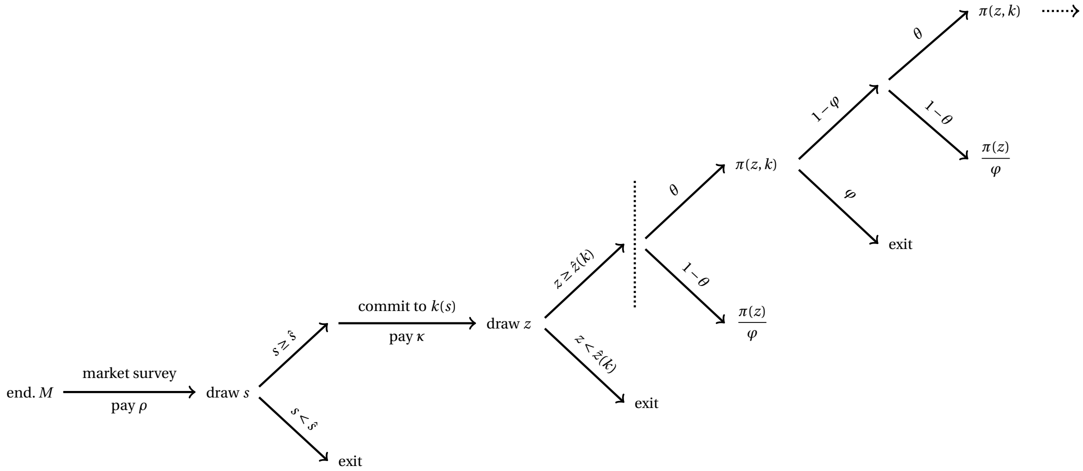

FIGURE 2. The graph above offers a diagrammatic representation of the sequence of decision: an endogenous mass of potential entrants runs a market survey and obtains a signal on their productive efficiency. If sufficiently optimistic, the resulting signal realization entails entry and translates into an optimally chosen initial capital stock. Upon learning of their actual productive efficiency realization, firms decide whether to take up production. The vertical dotted line marks the commencement of business as usual.

Profit maximisation. Conditional on operation and in conjunction with (2) and (3), profit maximization takes the form of a static intra-period optimization problem. In particular, the program of a firm that has already received its capital adjustment shock reads

$$\pi(z) = \max_{p,y,n,k} \left\{ py - Wn - Rk - \phi \;\middle|\; y = Yp^{-\sigma} \text{ and } y = \exp(z) n^{1-\alpha} k^{\alpha} \right\}. \qquad (5)$$

The optimal policies for output, capital demand, labor demand, and prices are, therefore, prescribed in terms of a univariate mapping from the productivity space Z to their respective co-domain.

An intermediate goods firm that is still bound by its initial capital commitment, on the other hand, solves

$$\pi(z,k) = \max_{p,y,n} \left\{ py - Wn - Rk - \phi \;\middle|\; y = Yp^{-\sigma} \text{ and } y = \exp(z) n^{1-\alpha} k^{\alpha} \right\} \tag{6}$$

for a fixed capital input $k \geq 0$. With the marginal cost pinned down by both z and k, the optimal policies for output, labor demand, and prices are defined on the Cartesian product $Z \times \mathbb{R}_+$; that is to say, as a function of both productive efficiency and initial capital commitment.

Taking up operations. An intermediate goods firm is fully characterized in terms of two random variables: its firm-level fundamental z as well as a noisy measurement thereof. Recognizing that the latter immediately translates into an optimally chosen initial capital commitment k[4], the value of a novice firm at the verge of taking up production is defined as[5]

$$v(z,k) = \sum_{a=0}^{\infty} \mathbb{E}\left[\pi_a \,\middle|\, z,k\right]. \tag{7}$$

Here, $\mathbb{E}[\pi_a \,|\, z,k]$ are the conditionally expected profits earned by a firm of age a with firm-level fundamental z and initial capital commitment k. That is,

$$\mathbb{E}\left[\pi_a \,\middle|\, z,k\right] = (1-\varphi)^a \left[\theta^{a+1}\tilde{\pi}(z,k) + \left(1-\theta^{a+1}\right)\pi(z)\right]. \tag{8}$$

As regards the timing of exit and capital adjustments shocks, a novice firm is guaranteed survival over its first period of operation while also being immediately eligible for a capital adjustment shock.

Evaluating the geometric series from (7), the value of taking up operations is then obtained as the survival-expectation of a convex combination of the lifetime profit flows for a *variable*- as well as a *fixed*-capital firm

$$v(z,k) = \frac{\lambda\pi(z) + (1-\lambda)\tilde{\pi}(z,k)}{\varphi} \quad \text{where} \quad \lambda = \frac{1-\theta}{1-\theta+\varphi\theta}. \tag{9}$$

Note that, λ is the fraction of equilibrium producers that have already received their capital adjustment shocks.

Since there is a fixed cost to production, it follows that every initial capital commitment $k \geq 0$ maps into a unique productivity threshold $\hat{z}(k) \in \operatorname{cl} Z$ such that firms with $(z,k) \,\middle|\, z < \hat{z}(k)$ do not find it worthwhile to produce. These considerations take

[4] Upon resolution of uncertainty, k is akin to a "sufficient stastic" for s.

[5] The exogenous exit rate $\varphi \in (0,1)$ obviates the need for an explicit account of a rate of time preference in order to ensure absolute summability. A discount factor $\beta \in (0,1)$ can be thought of as being absorbed by φ.

place vis-a-vis a fixed outside option.[6] In particular, with

$$\bar{K} = \sup\left\{ K \in \mathbb{R}_+ \,\middle|\, \lim_{z \to \sup Z} v(z,k) > 0 \ \forall \ k \le K \right\}$$

and recognizing that $v(z,k)$ is monotonically increasing in z for all $k \geq 0$, there is a well-defined productivity threshold $\hat{z} : \mathbb{R}_+ \to \mathrm{cl}\, Z$ prescribed by

$$k \mapsto \begin{cases} \inf\left\{ z \in Z \,\middle|\, v(z,k) \geq 0 \right\} & \text{if } k \leq \bar{K} \\ \sup Z & \text{otherwise.} \end{cases} \tag{10}$$

An intermediate goods firm with (z,k) optimally exercises its option of early exit, thereby realizing a payoff of zero, if and only if $z < \hat{z}(k)$. Figure 3 illustrates.

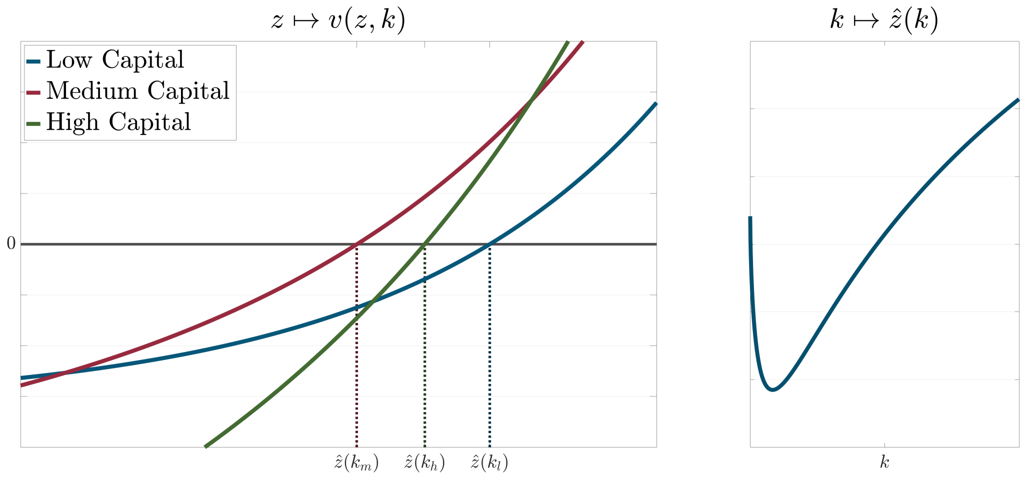

FIGURE 3. The lefthandside panel illustrates $z \mapsto v(z,k)$ for $k \in \{\text{low, medium, high}\}$. A low initial capital stock requires a high productive efficiency realization in order to make production worthwhile vis-a-vis the fixed cost of operation ϕ. A high initial capital stock, on the other hand, requires a high productive efficiency realization in order to recoup the correspondingly high user cost of capital. These considerations translate into the non-monotonicity observed in the righthandside panel.

Entry and capital commitment. The initial capital stock commitment of a potential entrant with signal realization s is pinned-down vis-a-vis their conditional expectation of $v(z,k)$ taking into account the option value of early exit. That is,

$$\tilde{k}(s) = \arg\max_{k \in \mathbb{R}_+} \int_{\hat{z}(k)} v(z,k)\, f(z|s)\, dz, \tag{11}$$

where

$$f(z|s) = \frac{f(z,s)}{\int_Z f(z,s)\, dz} \quad \text{and} \quad f(z,s) = h(z)\, g\left(\frac{s-z}{\tau}\right) \frac{1}{\tau}$$

[6] The fixed cost of production ϕ can be thought of a strictly positive outside option. Placing bounds on first derivatives, one could, in principle, entertain a capital-specific penalty for early exit.

such that $f : Z \times \mathbb{R} \to \mathbb{R}_+$ is obtained as the τ-scaled convolution of $h(z) = H(dz)/dz$ and $g(\xi) = G(d\xi)/d\xi$. Consequently, $f(\cdot|s)$ is the density of firm-level productive efficiency conditional on observation of a noisy measurement s as recovered via Bayes' rule. Naturally, $\tilde{k}(s)$ is monotonically increasing in s. The more optimistic the signal realization, the larger the initial capital commitment. Figure 4 illustrates.

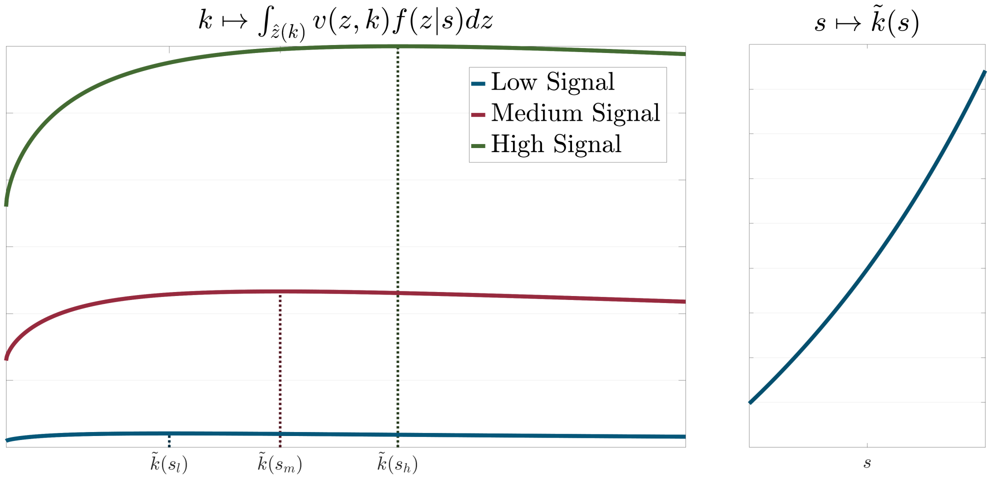

FIGURE 4. The mapping $k \mapsto \int_{\hat{z}(k)} v(z,k) f(z|s)\, dz$ is easily seen to be strictly concave for any given $s \in \mathbb{R}$. The lefthandside panel illustrates for some $s \in \{\text{low, medium, high}\}$. With the maximum shifting over to the right at s increases, the optimal choice of initial capital – as illustrated in the righthandside panel - is monotonically increasing in s.

Since entry entails the incurrence of a strictly positive entry cost κ, which is to say, since entry is costly by itself, a signal realization might be sufficiently pessimistic that a firm does not enter to begin with. In particular, $\exists!\ \hat{s} \in \mathbb{R}$ such that

$$\hat{s} = \inf\left\{ s \in \mathbb{R} \;\middle|\; \max_{k \in \mathbb{R}_+} \int_{\hat{z}(k)} v(z,k)\, f(z|s)\, dz \geq \kappa \right\}. \tag{12}$$

A potential entrant with a signal realization that falls short of $\hat{s}$ is, therefore, unwilling to bear the entry cost κ and decides to exit prior to the (costly) revelation of its fundamental. Figure 5 illustrates.

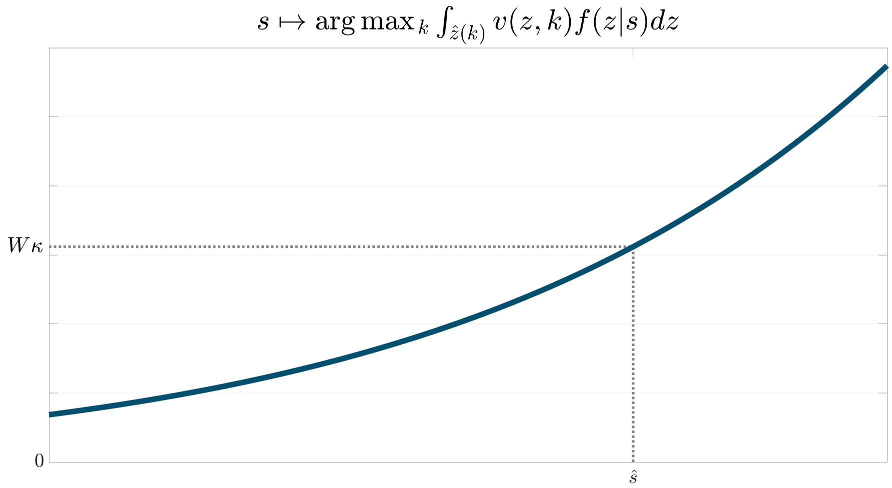

FIGURE 5. The graph above depicts the conditionally expected value of taking up operations accounting for the option value of early exit and positing an optimal choice of the initial capital stock. This value is plotted as a function of s. That is, $s \mapsto \mathbb{E}\big[\, v\big(z, \tilde{k}(s)\big)\mathbb{1}\{z \geq (\hat{z}\circ\tilde{k})(s)\} \,|\, s\big]$. Naturally, the image of $\mathbb{R}$ is a subset of the weakly positive real line with a infimum less that $W\kappa$. By first-order stochastic-dominance it thus follows that there is a unique $\hat{s}$.

Thinking about the induced sample space that underpins the distributional primitives in this model, it is straightforward to write down a predicate that formalizes the requisite for taking up operations

$$\mathscr{A} = \Big\{ s \geq \hat{s} \text{ and } z \geq (\hat{z}\circ\tilde{k})\,(s) \Big\}. \tag{13}$$

An intermediate goods firm with (z, s) enters *if and only if* their signal realization s exceeds the signal cutoff $\hat{s}$ and takes up production *only if* their fundamental realization z clears a productivity threshold $\hat{z}\circ\tilde{k}$ which is, itself, evaluated at the signal realization s. Figure 6 illustrates.

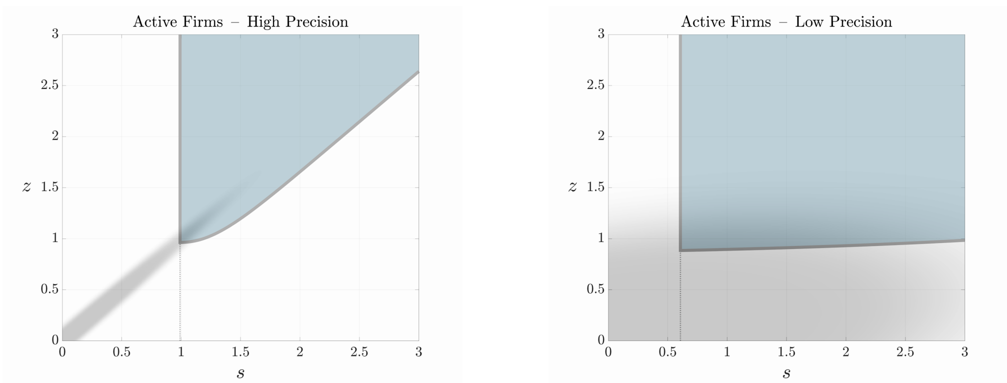

FIGURE 6. The lefthandside panel illustrates the demarcation of $\mathscr{A}$ in the (s, z)-space in a high precision environment (as it results in equilibrium). The gray shading corresponds to a birdseye-view on the joint density function of (s, z). A high precision environment is, naturally, characterized by an extensive amount of correlation. The pale-blue area marks the survival of selection. That is, $s \geq \hat{s}$ and $z \geq (\hat{z}\circ\tilde{k})(s)$. For the latter, higher s implies higher k which for any $s \geq \hat{s}$ requires a higher productivity to recoup capital cost. The righthandside panel depicts the selection-region in a low precision environment.

Potential entrants. The equilibrium mass of potential entrants is pinned down through an indifference condition. Specifically, indifference of the marginal potential entrant is predicated on the equation of the cost of running a market survey ρ and the unconditionally expected pay-off to running a market survey and behaving optimally afterwards. That is,

$$\int_{\hat{s}} \left(\max_k \int_{\hat{z}(k)} v(z,k)\, f(z|s)\, dz - \kappa \right) f(s)\, ds = \rho. \tag{14}$$

The economy. By way of summary, the economy at hand is defined in terms of a 12-tupel

$$\mathscr{E} = \left\langle \sigma, \alpha, \phi, \kappa, \rho, \tau, \varphi, \theta, R, L, H, G \right\rangle$$

where $\sigma > 1$ is the elasticity of substitution, $\alpha \in (0,1)$ is the elasticity of firm-level output w.r.t. capital, and $\phi, \kappa, \rho > 0$ denote the fixed cost of operation, entry cost, and information cost; all denominated in terms of final output. The parameter $\tau > 0$ governs the severity of information frictions, φ gives the rate of exogenous exit, and θ parameterizes the occurrence of capital adjustment shocks. The exogenous user cost of capital are given as R, while $L > 0$ pins down the inelastic labor supply. Finally, H and G are independent probability measures defined on the Borel σ-algebras of the log-productivity space Z and the real line, respectively. It is understood that $\int \xi G(d\xi) = 0$ and $\int \xi^2 G(d\xi) = 1$.

3.3 Equilibrium

Stationary equilibrium. Confining attention to a stationary equilibrium, entry and selection play out in a manner such that the mass of producers N is time-invariant. By the law of large numbers, each period a fraction φ of producers exits for exogenous reasons. Stationarity then requires that the set of equilibrium producers is replenished by the fraction of the mass of potential entrants M that survives that selection criteria formalized in terms of the Borel-measurable event $\mathscr{A}$. That is,

$$M\,\mathrm{Prob}(\mathscr{A}) = \varphi N. \tag{15}$$

The price index. With a CES aggregation technology, the price index is obtained as the usual dual[7]

$$\frac{(1-\lambda)M}{\varphi}\iint \mathbb{1}\{\mathscr{A}\}\,\tilde{p}(z,s)^{1-\sigma}f(z,s)ds\,dz+\frac{\lambda M}{\varphi}\iint \mathbb{1}\{\mathscr{A}\}p(z)^{1-\sigma}f(z,s)ds\,dz=1 \tag{16}$$

where P is normalized to unity due to the choice of final output as the economy's numeraire. Firm-level prices are simply a constant markup $\sigma/(\sigma-1)$ over marginal cost – which happen to differ in their computation depending on whether a firm is yet to receive its capital adjustment shock.

Labor market clearing. The same line of reasoning applies to labor market clearing. That is,

$$\frac{(1-\lambda)M}{\varphi}\iint \mathbb{1}\{\mathscr{A}\}\,\tilde{n}(z,s)f(z,s)ds\,dz+\frac{\lambda M}{\varphi}\iint \mathbb{1}\{\mathscr{A}\}n(z)f(z,s)ds\,dz=L. \tag{17}$$

Definition 1. A stationary competitive equilibrium for economy $\mathscr{E}$ is defined as

- A signal threshold $\hat{s}\in\mathbb{R}$.
- A productivity threshold $\hat{z}:\mathbb{R}_+\to \mathrm{cl}Z$.
- A mass of producers $N\geq 0$.
- A mass of potential entrants $M\geq 0$.
- An output level $Y\geq 0$.
- A wage rate $W\geq 0$.
- An initial capital commitment $\tilde{k}:\mathbb{R}\to\mathbb{R}_+$.
- Fixed-capital firm policies $\tilde{n}:Z\times\mathbb{R}\to\mathbb{R}_+$ and $\tilde{p}:Z\times\mathbb{R}\to\mathbb{R}_+$.
- Variable-capital firm policies $n:Z\to\mathbb{R}_+$, $k:Z\to\mathbb{R}_+$, and $p:Z\to\mathbb{R}_+$.

such that

(i) Taking as given W, Y, the prescription of initial capital stock $\tilde{k}$ satisfies (11).

(ii) Taking as given W, Y, fixed-capital firm policies are optimal such that $\tilde{n}$ and $\tilde{p}$ attain $\tilde{\pi}$ at $\tilde{k}$.

(iii) Taking as given W, Y, variable-capital firm policies are optimal such that n, k, and p attain π.

[7] A proper derivation of both the ecnonomy's price index as well as aggregate labor demand is deferred to Appendix A.

(iv) The marginal entrant's signal realization satisfies (12).

(v) The marginal potential entrants expects a net-payoff of zero as in (14).

(vi) The stationarity condition (15) holds true period-by-period.

(vii) The price index is given by (16).

(viii) The labour market clears according to (17).

A detailed discussion of the solution method is deferred to Appendix A.

3.4 Intuition from a model without capital

Under fairly innocuous regularity conditions on the distribution of primitives, my model offers a number of sharp predictions regarding the impact of a diminution of information frictions on selection, variety, and productive efficiency. For convenience of exposition, the subsequent discussion abstracts away from the usage of capital in the production of varieties. This is without much loss of generality. While certainly instrumental in ascertaining the severity of information frictions vis-a-vis the cross-sectional dispersion of marginal revenue products, rigidities in capital adjustments – or even the employment of capital itself – are not particularly consequential in shaping the model's selection mechanism. It is, therefore, assumed that $\theta \to 0$ and $\alpha = 0$. For the sake of notational convenience, it is also assumed that ϕ, κ, and ρ are denominated in units of labor rather than final output, while labor takes on the role of the economy's numeraire with the wage rate normalized to unity. This is merely in order to sidestep economically uninteresting feedback loops that affect efficiency results.[8]

A simplified model. Positing that production takes place with labor as its only input, profit flows $\pi(z)$ are as prescribed in (5). Since $\pi(z)$ runs from $\lim_{z\downarrow-\infty}\pi(z) = -\phi/\varphi$ and $\lim_{z\uparrow\infty}\pi(z) = \infty$ in a monotonically increasing fashion, there exists a null-measure of marginal producers indexed at a unique productivity-cutoff $\hat{z} \in \mathbb{R}$ such that

$$\pi(\hat{z}) = 0. \tag{18}$$

Upon resolution of uncertainty an entrant takes up production iff their flow profits are strictly positive, i.e. if and only if $z \geq \hat{z}$. Note that the survival-expectation of lifetime profits is given as $v(z) = \varphi^{-1}\pi(z)$. It follows that there is a marginal entrant,

[8]Without any loss of generality, in what follows I, moreover, set $Z = \mathbb{R}$.

indexed at a signal-cutoff $\hat{s} \in \mathbb{R}$, whose conditionally expected value $v(z)$ – taking into account the option of early exit – covers the entry cost κ by equality. That is,

$$\mathbb{E}\Big[\, v(z)\, \mathbb{1}\{z \geq \hat{z}\} \,\Big|\, \hat{s} \,\Big] = \kappa. \tag{19}$$

Once again, uniqueness and existence of $\hat{s}$ is guaranteed as the lefthandside expression (viewed as a function of $\hat{s}$) monotonically increases from 0 to ∞. Finally, the mass of potential entrants is pinned down through

$$\mathbb{E}\big[\, v(z)\, \mathbb{1}\{z \geq \hat{z}, s \geq \hat{s}\}\big] = \rho \tag{20}$$

where the lefthandside is to be understood as the unconditionally expected payoff of running a market survey and behaving optimally afterwards. The remaining equilibrium objects (Y, P, and M) are, then, pinned down through

$$PY = \sigma \phi \, \frac{M}{\varphi} \int_{\hat{z}} \int_{\hat{s}} \exp\big((\sigma-1)(z-\hat{z})\big)\, f(z,s)\, ds\, dz \tag{21}$$

$$P = \frac{\sigma}{\sigma-1} \left[\frac{M}{\varphi} \int_{\hat{z}} \int_{\hat{s}} \exp\big((\sigma-1)\, z\big)\, f(z,s)\, ds\, dz \right]^{\frac{1}{1-\sigma}} \tag{22}$$

$$PY = L \tag{23}$$

which is to say the profit functional, the CES price index, and goods market clearing, respectively.

Exploiting the recursive structure of the above system of equilibrium equations, it is easily seen that (18) to (20) encapsulate the entirety of model mechanisms pertaining to selection. Substituting out any dependence of $v(z)$ on aggregates, selection is, therefore, fully governed by two non-linear equations in $\hat{z}$ and $\hat{s}$. Namely, we have

$$A(\hat{z}, \hat{s}, \varsigma) = \int_{\hat{z}} \Big[\, e^{(\sigma-1)(z-\hat{z})} - 1 \,\Big]\, f(z|\hat{s}) dz - \frac{\kappa}{\phi} \tag{24}$$

and

$$B(\hat{z}, \hat{s}, \varsigma) = \int_{\hat{z}} \int_{\hat{s}} \Big[\, e^{(\sigma-1)(z-\hat{z})} - 1 \,\Big]\, f(z,s)\, ds\, dz - \frac{\kappa}{\phi} \iint_{\hat{s}} f(z,s)\, ds\, dz - \frac{\rho}{\phi}. \tag{25}$$

In order to set up notation for the subsequent discussion of existence and uniqueness, comparative statics, and efficiency properties, it proves instructive to define (and sign) the set of first-order partial derivatives of A and B with respect to $\hat{z}$ and $\hat{s}$.

Suppressing dependence on arguments, we have

$$
\begin{aligned}
A_{\hat{z}} &= \int_{\hat{z}} \frac{\partial}{\partial \hat{z}} \left[\mathrm{e}^{(\sigma-1)(z-\hat{z})} \right] f(z|\hat{s})\, dz \le 0 \\
A_{\hat{s}} &= \int_{\hat{z}} \left[\mathrm{e}^{(\sigma-1)(z-\hat{z})} - 1 \right] \frac{\partial f(z|\hat{s})}{\partial \hat{s}}\, dz \ge 0 \\
B_{\hat{z}} &= \int_{\hat{z}} \int_{\hat{s}} \frac{\partial}{\partial \hat{z}} \left[\mathrm{e}^{(\sigma-1)(z-\hat{z})} \right] f(z,s) \le 0 \\
B_{\hat{s}} &= \int_{\hat{z}} \left[1 - \mathrm{e}^{(\sigma-1)(z-\hat{z})} \right] f(z,\hat{s})\, dz + \frac{\kappa}{\phi} f(\hat{s}) = 0.
\end{aligned}
$$

The second inequality follows from a partial averaging notion of first-order stochastic dominance and the last equality is due to the fact that $A(\hat{z}, \hat{s}, \varsigma) = 0$ in equilibrium.

Existence and uniqueness. With equilibrium being pinned down in terms of two highly non-linear equations in two unknowns, existence and uniqueness are – generally speaking – not necessarily guaranteed. The following proposition establishes existence of a unique stationary equilibrium at any economically reasonable parameterization of $\mathscr{E}$.

Proposition 1. There exists a unique 2-tupel $(\hat{z}_*, \hat{s}_*) \in \mathbb{R}^2$ such that $A(\hat{z}_*, \hat{s}_*, \varsigma) = 0$ and $B(\hat{z}_*, \hat{s}_*, \varsigma) = 0$ for all $\rho, \kappa, \phi > 0$, $\tau > 0$, and $\sigma > 1$.[9]

Proof: Fix $\hat{z}_* \in \mathbb{R}_+$. Note that $\lim_{\hat{s}\downarrow-\infty} \lim_{\hat{z}\downarrow 0} A(\hat{z}, \hat{s}) = \lim_{\hat{s}\uparrow\infty} \lim_{\hat{z}\downarrow 0} A(\hat{z}, \hat{s}) = \infty$ while $\lim_{\hat{s}\downarrow-\infty} \lim_{\hat{z}\uparrow\infty} A(\hat{z}, \hat{s}) = \lim_{\hat{s}\uparrow\infty} \lim_{\hat{z}\uparrow\infty} A(\hat{z}, \hat{s}) = -\kappa/\phi$. It follows that $A_{\hat{z}} \le 0$ and $A_{\hat{s}} \ge 0 \implies \exists!\ \hat{s}_* \in \mathbb{R} : A(\hat{z}_*, \hat{s}_*) = 0$. Prescribe a bijection $\hat{s}_* : \mathbb{R}_+ \to \mathbb{R}$ accordingly and note that $\hat{s}_*(x') \ge \hat{s}_*(x)$ for all $x' > x$. Now, we have $B_{\hat{z}} \le 0$ and $B_{\hat{s}}(\hat{z}, \hat{s}_*(\hat{z})) = 0$ $\forall\ \hat{z} \in \mathbb{R}_+$. Finally, $\lim_{\hat{z}\downarrow 0} B(\hat{z}, \hat{s}_*(\hat{z})) = \infty$ while $\lim_{\hat{z}\uparrow\infty} B(\hat{z}, \hat{s}_*(\hat{z})) = -\rho/\phi$. Therefore, $\exists!\ \hat{z}_* : B(\hat{z}_*, \hat{s}_*(\hat{z}_*)) = 0$. ■

Corollary to Proposition 1. There exists a unique stationary market equilibrium for economy $\mathscr{E}\big|_{\alpha=0}$.

Proof: Conditional on $\hat{z}$ and $\hat{s}$, the set of equations in (21) to (23) allows for the recovery of unique (real) closed-form solution of Y, P, and M. In particular, we have

$$Y = \psi L (\sigma - 1) \exp(\hat{z}) \tag{26}$$

$$P = \psi^{-1} (\sigma - 1)^{-1} \exp(-\hat{z}) \tag{27}$$

$$M = \frac{\varphi}{\sigma \phi} L \left[\int_{\hat{z}} \int_{\hat{s}} \exp\big((\sigma - 1)(z - \hat{z}) \big) f(z,s)\, ds\, dz \right]^{-1} \tag{28}$$

[9] The dependence of A and B on ς is henceforth suppressed for notational convenience.

where $\psi = \sigma^{-\frac{\sigma}{\sigma-1}} \phi^{-\frac{1}{\sigma-1}} L^{\frac{1}{\sigma-1}}$. Uniqueness of the equilibrium for $\mathscr{E}\big|_{\alpha=0}$ consequently follows from uniqueness of $\hat{z}$ and $\hat{s}$ as established in Proposition 1.

■

Comparative statics. The following subsection ascertains the directionality of movements in equilibrium quantities due to changes in the structural parameterization of $\mathscr{E}$. Particular emphasis is placed on the implications of a movement towards a higher precision environment, i.e. a decrease in τ.

Equilibrium dictates that $A(\hat{z}, \hat{s}, \varsigma) \equiv 0$ and $B(\hat{z}, \hat{s}, \varsigma) \equiv 0$ at every $\varsigma = \langle \tau, \sigma, \kappa/\phi, \rho/\phi, \eta \rangle$. Total differentiation of A and B with respect to ς, therefore, yields a linear system of two equations in $d\hat{z}/d\varsigma$ and $d\hat{s}/d\varsigma$ the solution of which is given by

$$\frac{d\hat{z}}{d\varsigma} = -\frac{B_\varsigma}{B_{\hat{z}}} \qquad \text{and} \qquad \frac{d\hat{s}}{d\varsigma} = -\frac{A_\varsigma}{A_{\hat{s}}} - \frac{A_{\hat{z}}}{A_{\hat{s}}} \frac{d\hat{z}}{d\varsigma}. \tag{29}$$

Proposition 2. Let $g(\xi) \propto g'(\xi)\,\xi$. It then follows that the productivity-cutoff $\hat{z}$ decreases as information frictions grow more severe. That is,

$$\frac{d\hat{z}}{d\tau} < 0.$$

Proof: The proof of Proposition 2 is deferred to Appendix B.

In order to build intuition for the comparative static from Proposition 2, it proves instructive to impose an auxiliary (imagined) sense of sequentiality on the movements of aggregates. Starting from a comparatively low precision environment,[10] i.e. with $\tau \gg 0$, a *ceteris paribus* increase in τ does not have any impact on the marginal producer. This is easily seen bringing to mind that, upon the resolution of uncertainty and against the background of fixed aggregates, a change in τ is entirely immaterial in pinning down profits of an individual producer. The marginal entrant, however, is bound to appreciate a lessening of the precision of their signal. As will be established with Proposition 3, a high precision environment entails a comparatively lax selection process. The corresponding signal-cutoff $\hat{s}$, therefore, does not bode well for production. With a loss of precision, the marginal entrant has reason to hope for a more auspicious productivity realization. As a consequence, their conditionally

[10]There is more nuance to the argument in a high precision environment. The principal forces are, however, the same.

expected value of entry net of entry cost becomes strictly positive; the equilibrium signal-cutoff decreases.

Since the cost of information are fixed, increasing τ is tantamount to saying that would-be potential entrants are free to purchase a lower quality signal without enjoying any decrease in information cost. The marginal potential entrant walks away and demand is, therefore, bound to become stronger (via an increase in YP^{σ}) when moving towards a lower precision environment. With a strengthening of demand, firm-level profits are higher at any given level of z, which implies that equilibrium sustains firms of overall lower productive efficiency. The productivity-cutoff decreases as established in Proposition 2.

Note that, having normalizes the economy's wage rate to unity, real wages are given as $\tilde{W} = P^{-1}$. Aggregate TFP is, moreover, pinned down as

$$\tilde{Z} = \left[\frac{M}{\varphi} \int_{\hat{z}} \int_{\hat{s}} \exp\left((\sigma - 1)\, z \right) f(z,s)\, ds\, dz \right]^{\frac{1}{\sigma-1}} = \psi\, \sigma \exp(\hat{z}).$$

Corollary to Proposition 2. Consumption-equivalent welfare, real wages, and aggregate TFP are decreasing as information frictions grow more severe. That is,

$$\frac{dY}{d\tau} < 0, \quad \frac{d\tilde{W}}{d\tau} < 0, \quad \text{and} \quad \frac{d\tilde{Z}}{d\tau} < 0.$$

Proof: Since Y, $\tilde{W}$, and $\tilde{Z}$ are upward-sloping linear functions of $\exp(\hat{z})$, their comparative statics inherit their directionality from $d\hat{z}/d\varsigma$.

■

Proposition 3. Selection becomes less stringent as information frictions grow more severe. That is

$$\frac{d\text{Prob}(s \geq \hat{s} \text{ and } z \geq \hat{z})}{d\tau} > 0 \quad \text{and} \quad \frac{d\text{Prob}(s \geq \hat{s})}{d\tau} > 0.$$

Even though the prediction in Proposition 3 appears to be true across an extensive variety of distributional assumptions and parameterizations, its more general proof seems rather elusive (still work in progress).

Intuitively speaking, as τ approaches zero, only producers populating the very tails of the productivity distribution enter to begin with. Moreover, since the observation of one's private signal is virtually tantamount to the resolution of uncertainty potential entrants enter iff the are also going to take up production. Selection plays out in as strict a manner as possible. On the other hand, as information frictions grow more severe, even a pessimistic signal realization leaves plenty of probability mass on favorable productivity realizations. In *reductio ad absurdum*, as τ tends to infinity, would-be potential entrants anticipate conditioning on trivial σ-algebra. It follow that a willingness to incur the cost associated with the purchase of an entirely uninformative signal implies the a willingness to also incur the entry cost – irrespective of the signal realization.

Proposition 4. The equilibrium mass of potential entrants decreases as information frictions grow more severe. That is,

$$\frac{dM}{d\tau} < 0$$

Proof: The proof of Proposition 4 is deferred to Appendix B. Figure 7 below illustrates.

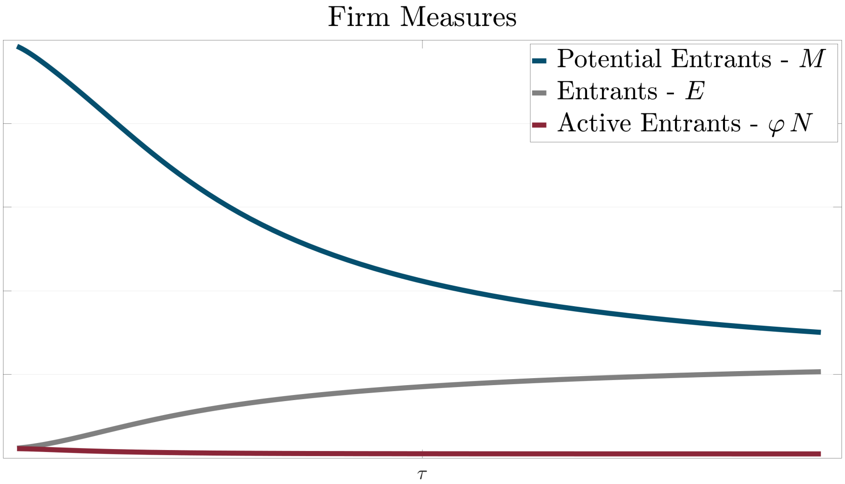

FIGURE 7. The graph above depicts a set of comparative statics for M, E, and φN. As shown in Proposition 4, the measure of potential entrants is monotonically decrease in τ. Proposition 3 implies that as information becomes infinite precise $\varphi N \uparrow E$. On the other hand, as $\tau \to \infty$ we have that $E \uparrow M$.

Efficiency properties. This section establishes that both the acquisition as well as utilization of information in the market equilibrium for $\mathscr{E}\big|_{\alpha=0}$ is perfectly consistent with the prescription of a benevolent social planner. This result is very much in line with Bilbiie, Ghironi, and Melitz (2006) as well as, more recently and in a more similar setting, Dhingra and Morrow (2019).

Proposition 5. The market equilibrium is efficient iff aggregation takes place according to a power-function.

Proof: The proof of Proposition 5 is deferred to Appendix B.

The implication of Proposition 5 is that the resources "wasted" on firms that ultimately do not take up production and firms that ultimately do not enter are a necessary evil in order to bolster selection. In particular, there are two principal channels for selection to affect aggregate TFP: *variety* and *productive efficiency*. Since selection is more stringent in a high precision environment, the set of equilibrium producers - irrespective of its measure - operates at a higher level of productive efficiency. Equilibrium, however, also attracts larger mass of potential entrants. So much so that, even though selection is more exacting, the measure of equilibrium producers increases. A diminution of information frictions entails an increase in the measure of varieties marketed in equilibrium as well as an increase in the productive efficiency of the firms producing these varieties. Both of those effects are bound to boost TFP which translates into the welfare-gains established in the Corollary to Proposition 2.

4 Calibration & model predictions

In this section, I outline my calibration strategy. Building on the calibrated version of my model, I then discuss the welfare-gains to be had from a hypothetical elimination of the entirety of entry-stage information frictions as well as the impact of selection on sales concentration.

When it comes to calibration, there are two model-objects that prove particularly informative in pinning down both price (ρ) and quality (τ) of information. Specifically, I leverage the information content of both *exit rates* among comparatively young establishments as well as age-profiles in cross-sectional *MRPK dispersion*. Intuitively, a decline in early exit speaks to the fact that entry decisions are made in a more informed manner, while a decline in factor-misallocation speaks to the availability of more precise forecasts when striking up initial capital commitments.

Failed entry. Whenever a firm relinquishes production within, say, their first year of operations, this should be taken as highly suggestive of a entry-stage misjudgment of their own productive efficiency and/or idiosyncratic demand shifts. In the confines of my model, one is bound to deduce that the firm's entry-stage forecast of z turned out rather poorly. The overall exit-rate among comparatively young establishments should, therefore, speak to the severity of information frictions in a very pointed manner. Turning to a corresponding comparative static in τ, Figure 8 illustrates the measure of firms that enter based on their signal realization and immediately decide to exit upon learning of their actual productive efficiency (as a fraction of the mass of entrants).

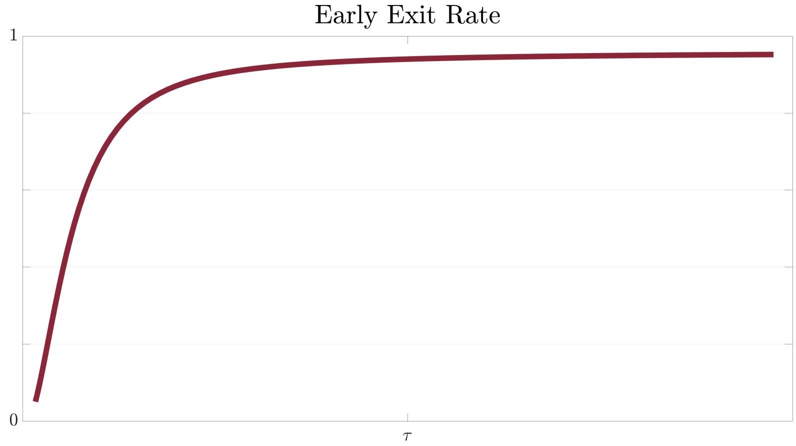

FIGURE 8. The graph above depicts a comparative static for $(E-\varphi N)/E$ in τ. As information become infinitely precise $\text{Prob}(z \geq \hat{z} \circ \tilde{k}(s) \,|\, s \geq \hat{s}) \to 1$ such that $\varphi N \uparrow E$.

Naturally, as τ approaches zero, the observation of a private signal on one's fundamental realization is very much akin to the resolution of uncertainty. As a consequence, only firms that are ultimately going to end up operating enter to begin with and the rate of early exit goes down to zero.

Cross-sectional MRPK dispersion. Painting a, perhaps somewhat more nuanced picture, cross-sectional dispersion in the *marginal revenue product of capital* speaks to factor-misallocation as it results from the commitment to an initial capital stock under imperfect information. There are two types of firm operating in equilibrium. For those who have already received their capital adjustment shock, we have

$$MRPK_i\big|_V = \frac{\alpha}{1+\mu}\,\frac{p(z_i)\,y(z_i)}{k(z_i)} = R.$$

Here, MRPK is merely a constant equal to the user cost of capital. For firms that are still bound by their initial capital commitment, however, we have

$$MRPK_i\big|_F = \frac{\alpha}{1+\mu}\,\frac{\tilde{p}(z_i,s_i)\,\tilde{y}(z_i,s_i)}{\tilde{k}(s_i)} \in \mathscr{L}^2$$

which is a non-degenerate and, under fairly mild regularity conditions, square-integrable random variable. It, thus, follows that

$$\text{Var}\big(MRPK_i\big) = \int_V \big(MRPK_i - R\big)^2\,di + \int_F \big(MRPK_i - R\big)^2\,di > 0$$

almost surely.[11] Confining oneself to a particular age-cohort, i.e. a particular $a \geq 0$, the corresponding age-specific cross-sectional MRPK dispersion is computed as the square-root of

$$MRPKD_a = \theta^{a+1} \left[\iint \left(\frac{\alpha}{1+\mu} \frac{\tilde{p}(z_i, s_i)\, \tilde{y}(z_i, s_i)}{\tilde{k}(s_i)} \right)^2 f(z, s \,|\, \mathscr{A})\, ds\, dz - R^2 \right]. \tag{30}$$

Looking at a comparative static, as depicted in Figure 9, cross-sectional MRPK dispersion is increasing as information becomes less precise.

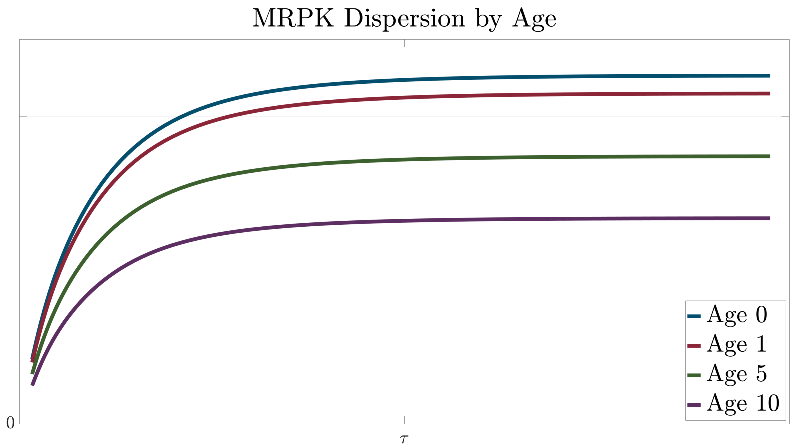

FIGURE 9. The graph above depicts a set of comparative statics for cross-sectional MRKP dispersion in τ. Each comparative static is obtained confining computation to a single age-cohort.

The less precise the forecast of z_i, the larger the discrepancy between $k(z_i)$ and $\tilde{k}(s_i)$ and the more a firm is going to compensate for their sub-optimal choice of capital via their employment of labor. As a result, the expected distance between the marginal revenue product of capital and R is bound to increase as information frictions grow more severe. Figure 9 does, however, also betray a gradient along the dimension of age. The older a firm, the more like they are to have received their capital adjustment shock, which is tantamount to saying that the fraction of firms with $MRPK_i = R$ increases with age.

Calibration. The comparative statics discussed in the preceding paragraphs set the stage for a relatively straightforward calibration exercise. For tangibility, I assume a jointly normal distribution for z and ξ. Maintaining the assumption that ξ takes on the role of the unit-variance noise component in (4), let the marginal distribution for the firm-level fundamental be given as $z \sim \mathcal{N}(0, \eta^2)$. The standard deviation

[11] The first term in the above equation is identically equal to zero.

of z does, therefore, unilaterally parameterize the latent productivity distribution. Given the symmetry properties of the normal distribution and keeping in mind the option value of early exit, η is most instructively though of as being reflective of the economy's technological status-quo.[12]

Parameters that are ultimately inconsequential for the key mechanism of model framework are simply assigned standard values that place $\mathscr{E}$ in an economically reasonable region of the parameter space. In particular, the elasticity of substitution σ is chosen to match a mid-range estimate for the aggregate markup of roughly 14%. The output elasticity of capital α is set to its conventional value of about 33%. Taking a model period to be equal to one year, the user cost of capital R is assigned a value of 10% reflective of 6% depreciation and 4% interest *per annum*. The exit rate φ is chosen to match the employment share of exiting firms and taken from Boar and Midrigan (2020). The Bernoulli rate of a capital adjustment shock is, in principle, identified by averaging over consecutive estimates for $MRPKD_{a+1}/MRPKD_a$. Without access to micro-data on revenue and capital stock – or age-specific statics for MRPK dispersion[13] – the recovery of a reasonable value for θ is deferred to the joint calibration exercise delineated below. Finally, the fixed cost of production ϕ and the measure of inelastically supplied units of labor L are chosen to normalize Y and N to be equal to unity.

Parameter	Speaks to:	Target/Assignment
τ	severity of information frictions	MRPK dispersion (0.0381)
ρ	cost of information	early exit (0.16)
η	technology parameter	sales share top 5% (0.57)
κ	entry barriers	entry rate (0.21)
σ	elasticity of substitution	aggregate markup (0.14)
α	output elasticity w.r.t capital	capital share in production (0.33)
φ	probability of exit shock	average exit-rate (0.04)
θ	counter-probability of capital adj.	age-profile in MRPK dispersion (0.82)
R	user-cost of capital	rental rate & depreciation (0.10)
ϕ	overhead	normalization ($N = 1$)
L	inelastic labor supply	normalization ($Y = 1$)

TABLE 2. Heuristic guidance of joint calibration exercise for (τ,ρ,η,κ). The remaining parameters are either assigned based on standard value in the literature or calibrated to normalize equilibrium quantities.

The joint calibration exercise concerns both quality and price of information, the parameterization of the latent productivity distribution, as well as the economy's entry

[12]For instance, see De Loecker, Eeckhout, Mongey (2021).

[13]Once I have been granted access to the 8$^{\text{th}}$ vintage of CompNet data, I am going to implement this much more convenient assignment protocol.

barriers. Heuristically speaking, the quality of information τ is chosen targeting the level of MRPK dispersion in US manufacturing documented by Kehrig and Vincent (2017). Similarly, given the data availability issues broached in the preceding paragraph, θ is – somewhat crudely – pinned down via MRPL dispersion. The cost of information ρ are chosen to match an early exit rate of about 16% while the entry cost κ target an entry rate of 21%. Both of those figures are based on BDS data for 2012 US manufacturing. Finally, the parameterization of the latent productivity distribution targets the sales share of the top 5% of firms in 2012 US manufacturing – specifically the average sales share by industry market share.[14]

The cross-sectional TFPQ distribution. With selection driving a refinement of the set of equilibrium producers, the cross-sectional distribution of firm-level fundamentals conditional on operating is obtained as an equilibrium object to the effect that

$$\text{Prob}\big(z_i \in dz \,\big|\, i \in N\big) = \left[\int_{\hat{s}} h(z) g\left(\frac{s-z}{\tau}\right)\frac{1}{\tau}\frac{\mathbb{1}\{\mathscr{A}\}}{\text{Prob}(\mathscr{A})}\, ds\right] dz$$

where $\mathscr{A}$ is as prescribed in (13). The graph in Figure 10 does, in that vein, depict the distribution of productive efficiency among firms that survived selection.

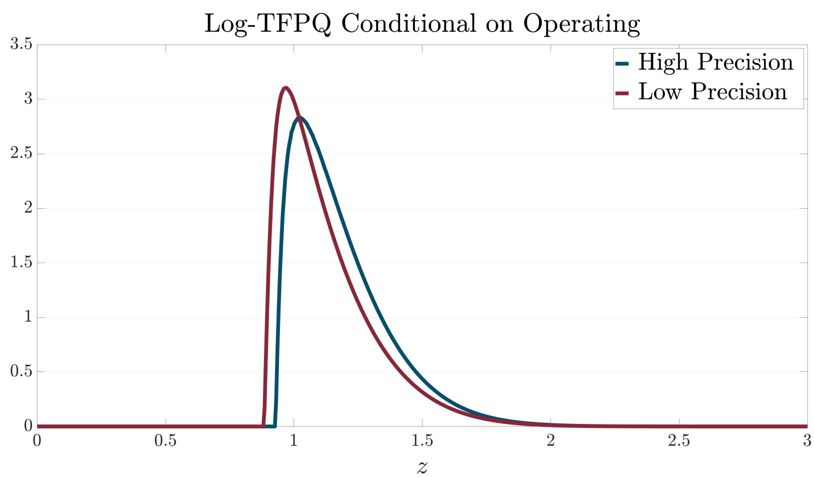

FIGURE 10. The graph above depicts the distribution of firm-level productive efficiency conditional on the survival of selection. That is, $z \mapsto \int_{\hat{s}} f(z,s|\mathscr{A})\, ds$ for $\tau \in \{\text{low, high}\}$.

The illustration above is confined to contrasting said productivity distribution in a high precision as well as a low precision environment. As will be corroborated with the subsequent discussion of market concentration, positing a diminution of information frictions over time, the model framework predicts a reallocation of sales from

[14]The figure of 0.57 is taken from Edmond, Midrigan, and Xu (2021).

low productivity to high productivity firms. The productivity distribution in a high precision environment features a higher mean and a higher "lower-bound" on its support, but also a considerably higher level of dispersion and an increase in interpercentile ranges. Against this backdrop, and keeping in mind that the equilibrium measure of producers is bound to increase as information becomes more precise, a decrease in τ over time is perfectly consistent with a number of well-documented results and salient features of the data. Baqaee and Fahri (2020) do, for example, argue that aggregate TFP growth has been largely driven by improvements in allocative efficiency. Akcigit and Ates (2021) document an increase in the productivity distance between frontier and laggard firms.

Welfare implications. This paper being geared towards an assessment of the welfare cost associated with entry-stage information frictions, Figure 11 below depicts the comparative static for consumption-equivalent welfare as in obtains upon quantification of my model. The vertical dotted line is to indicate the severity of information frictions as ascertained in the above calibration exercise.

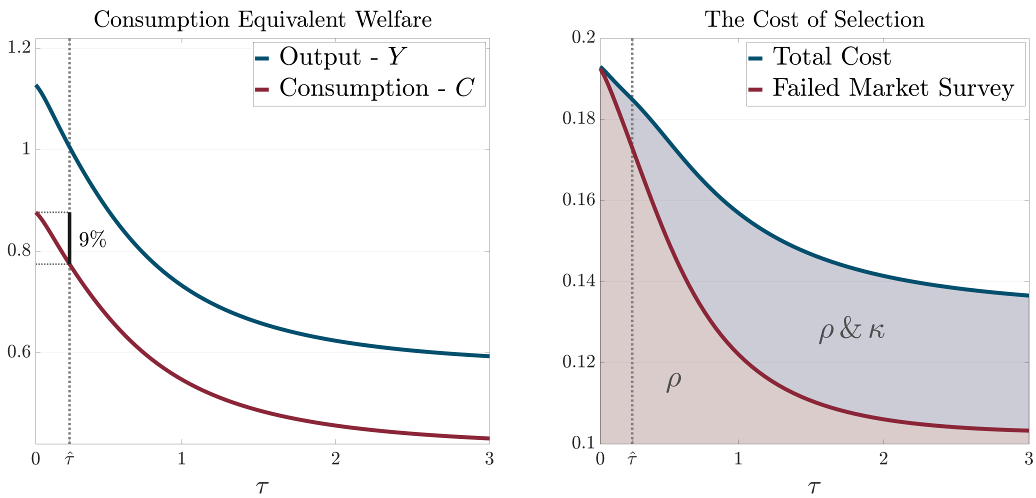

FIGURE 11. The lefthandside depicts a comparative static for output Y as well as consumption-equivalent welfare C. Also in terms of a comparative static, the righthandside panel illustrates the resources spent on failed market surveys (firms that incur $W\rho$ in order to purchase a signal but subsequently do not enter) as well as failed entry (firms that enter incur $W\rho$ and $W\kappa$ but do not take up production).

The welfare-gains to be had from a hypothetical elimination of the entirety of entry-stage information frictions are in a ballpark of roughly 10%. It is to be noted that the only source of inefficiency in this model is due to the capital supply margin. Having adopted a monopolistically competitive market structure alongside a CES demand system, pricing decisions always entail the extraction of rents. Firms are, broadly speaking, too small. With labor being supplied inelastically there is no margin for distortion, whatsoever. Capital is, however, supplied in an infinitely elastic manner such that the presence of markups creates a wedge that distorts the aggregate employment of capital in production. Arguing along the lines of Proposition 5, when counteracting this particular inefficiency, both the acquisition and utilization of information turn out to be perfectly consistent with the prescription of a social planner.[15]

The righthandside panel in Figure 11 illustrates the resources "wasted" on entrants that immediately exit of their own volition (blue) as well as potential entrants whose signal discourages entry (red). As information becomes more precise, selection becomes more costly with the bulk of expenditures accruing in the form of information cost.[16]

Concentration. Complementing the perspective assumed in Eeckhout and Veldkamp (2022), I use my model-framework as a lens in order to connect the severity of information frictions to sales concentration – the latter being measured as the revenue share of the top 5% of firms operating in equilibrium. In order to compute the corresponding model object, let $\tilde{N}$ denote the subset of equilibrium producers whose revenue clears the 95$^{\text{th}}$ percentile of the revenue distribution. Partitioning $\tilde{N}$ according to whether a firm is still bound by its initial capital commitment, the relevant revenue share is then obtained as

$$RS_{5\%} = \int_{\tilde{N}} \frac{r_i}{Y} \, di = \int_{\tilde{F}} \frac{r_i}{Y} \, di + \int_{\tilde{V}} \frac{r_i}{Y} \, di$$

where

$$r_i \big|_{\tilde{F}} = \tilde{r}(z_i, s_i) = \tilde{p}(z_i, s_i) \, \tilde{y}(z_i, s_i) \qquad \text{and} \qquad r_i \big|_{\tilde{V}} = r(z_i) = p(z_i) \, y(z_i).$$

In order to demarcate $\tilde{N}$ in the (s, z) - space, it proves instructive to define an iso-revenue curve $\tilde{z}_{r_*}(s) : \tilde{r}\big(\tilde{z}_{r_*}(s), s\big) = r_*$ where r_* is the 95$^{\text{th}}$ percentile of the revenue distribution. The level of $\tilde{z}_{r_*}(s)$ is pinned down recovering the unique scalar $\tilde{z} \in \mathbb{R}$ that satisfies

$$\lambda \, \text{Prob}\big(\{z \geq \tilde{z}\} \cup \mathscr{A}\big) + (1 - \lambda) \, \text{Prob}\big(\{z \geq \tilde{z}_{r(\tilde{z})}(s)\} \cup \mathscr{A}\big) = 5\%.$$

such that $r(\tilde{z}) = r_*$. Note that

$$r(\tilde{z}) = \tilde{r}(\tilde{z}_{r(\tilde{z})}(s), s) \quad \Longleftrightarrow \quad \inf \tilde{r}\big(\partial\big\{(z, s) : \{z \geq \tilde{z}_{r(\tilde{z})}(s)\} \cup \mathscr{A}\big\}\big) = r(\tilde{z}).$$

[15]This statement regarding efficiency also warrants an abstraction from feedback loops created through the denomination of ϕ, κ, and ρ in units of final good.

[16]See the discussion of Propositions 3 & 4 for intuition.

Figure 12 illustrates.

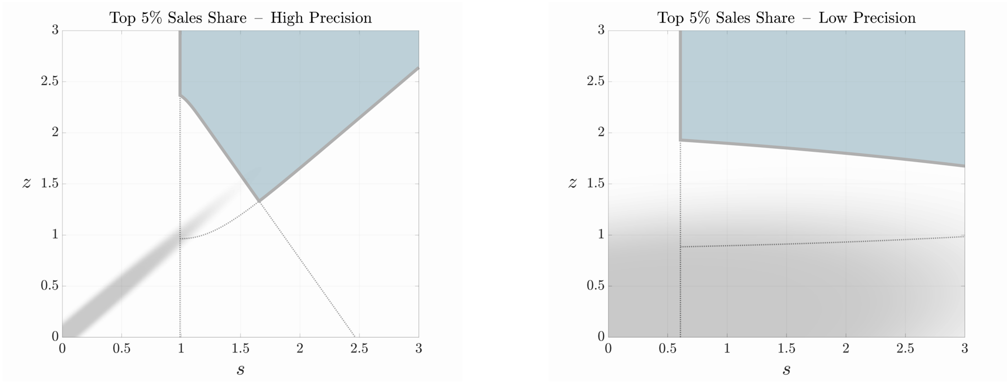

FIGURE 12. The lefthandside panel illustrates the demarcation of $\tilde{N}$ in the (s, z)-space in a high precision environment. The gray shading corresponds to a birdseye-view on the joint density function of (s, z). The pale-blue area corresponds to the conditions on (s, z) that imply as revenue above the 95$^{\text{th}}$ percentile of the revenue distribution. That is, $s \geq \hat{s}$ and $z \geq \max\left\{(\hat{z} \circ \tilde{k})(s), \tilde{z}_{r(\tilde{z})}(s)\right\}$ where the dotted lines respectively correspond to $\hat{s}$ (vertical), $\hat{z} \circ \tilde{k}$ (upward-sloping), and $\tilde{z}_{r(\tilde{z})}$ (downward-sloping). The righthandside panel depicts the same demarcation in a low precision environment.

Figure 13, moreover, illustrates a comparative static for $RS_{5\%}$ in τ.

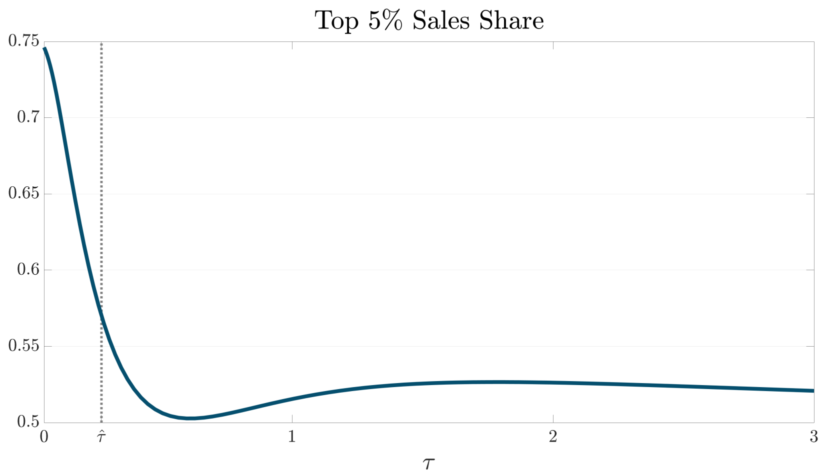

FIGURE 10. The graph above depicts the (calibrated) comparative static for the top 5% sales-share in τ.

Once the economy enters a comparatively high precision region of the parameter space, concentration increases in precision – and quite responsively at that. A hypothetical elimination of the entirety of entry-stage information frictions would see around 75% of sales in the hand of the top 5% revenue strongest firms.

5 Conclusion

I study the welfare-implications of a diminution of entry-stage information frictions in the US economy. Corroborating that the past few decades have indeed seen an abatement of information frictions, I provide reduced-form evidence that supports a marked improvement in private-sector forecasting practices. Specifically, I document that the evolution of forecast-specific language in 10-K filings reflects the adoption of evermore sophisticated forecast protocol. The fact that these patterns are even more pronounced for firms approaching IPO lends credibility to the transfer and extrapolation of these trends to establishments at the verge of entry. Finally, correlating cross-sectional differentials in key-performance indices with forecast attentiveness bolsters the assuagement of cheap talk concerns.

In order to explore the welfare-implication of this continuing improvement in the quality of private sector forecasts, I entertain a simple Chamberlinian model with entry-stage selection and rigidities in capital adjustments. I show that, as information becomes more precise, selection becomes more stringent. This theoretical result holds true under entirely innocuous distributional assumptions. An increasingly selective refinement of the set of equilibrium producers does, in principle, set the stage for a tension between love-of-variety and productive efficiency. As selection becomes more exacting, equilibrium producers operate at a higher level of productive efficiency, but a lower fraction of (potential) entrants survives. Obviating the concern of a dwindling measure of varieties, I show that, as information becomes more precise, equilibrium also attracts a larger mass of potential entrants; so much so that the mass of equilibrium producers goes up despite the increase in selectivity. The interplay of attraction and selection, therefore, boosts aggregate TFP via both the *variety* as well as the *productive efficiency* channel. Quantifying the impact of TFP-gains on consumption-equivalent welfare, I find that welfare-gains to be had from the hypothetical elimination of entry-stage information frictions in the US economy are in the ballpark of 10%. As regards the efficiency properties of my model, both the acquisition as well as the utilization of information play out in a perfectly efficient manner. In the relevant region of the parameter space, sales-concentration is shown to increase in precision.

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Appendix A

Derivation of CES Price-Index

The price index dual to the CES aggregation technology assumed for $\mathscr{E}$ is defined as

$$P^{1-\sigma} = \int_N p_i^{1-\sigma}\, di.$$

The economy features two types of firms collected in the set of equilibrium producers N. In particular, we have firms that have already received their capital adjustment shock and firms that are still bound by their initial capital commitment. Partitioning the set of equilibrium producers accordingly, we have

$$F = \left\{ i \in N \,\middle|\, k_i = \tilde{k}(s_i) \right\} \qquad \text{and} \qquad V = \left\{ i \in N \,\middle|\, k_i = k(z_i) \right\}$$

such that $F \cup V = N$ and $F \cap V = \emptyset$. Since the Lebesgue-integral is additive in partitions of the integration domain, it follows that

$$P^{1-\sigma} = \int_F p_i^{1-\sigma}\, di + \int_V p_i^{1-\sigma}\, di.$$

At this point, it proves instructive to note that

$$p_i\big|_F = \tilde{p}\big(z_i, \tilde{k}(s_i)\big) \qquad \text{while} \qquad p_i\big|_V = p(z_i).$$

With conventional abuse of notation that e.g. F denotes both the set of fixed-capital producers as well as its measure, a straightforward change of measure yields

$$P^{1-\sigma} = F \int\int \tilde{p}\big(z, \tilde{k}(s)\big)^{1-\sigma} F(dz, ds\,|\,\mathscr{A}) + V \int\int p(z)^{1-\sigma} F(dz, ds\,|\,\mathscr{A}).$$

Finally, since both exit as well as capital adjustment shocks are entirely orthogonal to firm-level fundamentals, we can easily compute the fraction of fixed-capital firms in our set of equilibrium producers. That is, with

Age	Fixed k	Mass
Age 0	θ^1	$(1-\varphi)^0 \varphi N$
Age 1	θ^2	$(1-\varphi)^1 \varphi N$
Age 2	θ^3	$(1-\varphi)^2 \varphi N$
$\vdots$	$\vdots$	$\vdots$

it is straightforward to see that

$$F = \frac{\theta\varphi}{1-\theta+\varphi\theta} N \qquad \text{while} \qquad V = \frac{1-\theta}{1-\theta+\varphi\theta} N.$$

So with $\lambda = (1-\theta)/(1-\theta+\varphi\theta)$ we have

$$(1-\lambda)\, N \int\int \tilde{p}\big(z, \tilde{k}(s)\big)^{1-\sigma} F(dz, ds \,|\, \mathscr{A}) + \lambda\, N \int\int p(z)^{1-\sigma} F(dz, ds \,|\, \mathscr{A}).$$

With the stationarity condition from (15) we are therefore, finally, to conclude that

$$P^{1-\sigma} = \frac{(1-\lambda)\, M \operatorname{Prob}(\mathscr{A})}{\varphi} \iint \tilde{p}\big(z, \tilde{k}(s)\big)^{1-\sigma} F(dz, ds \,|\, \mathscr{A}) + \frac{\lambda\, M \operatorname{Prob}(\mathscr{A})}{\varphi} \iint p(z)^{1-\sigma} F(dz, ds \,|\, \mathscr{A}).$$

Now, the final thing to notice is that

$$F(dz, ds \,|\, \mathscr{A}) = \frac{\mathbb{1}\{\mathscr{A}\}\, F(dz, ds)}{\operatorname{Prob}(\mathscr{A})}.$$

Hence,

$$P^{1-\sigma} = \frac{(1-\lambda)\, M}{\varphi} \iint \mathbb{1}\{\mathscr{A}\}\, \tilde{p}\big(z, \tilde{k}(s)\big)^{1-\sigma} F(dz, ds) + \frac{\lambda\, M}{\varphi} \iint \mathbb{1}\{\mathscr{A}\} p(z)^{1-\sigma} F(dz, ds).$$

The derivation of the labor market clearing condition is perfectly analogous.

Solution method

We have a set of 5 equilibrium equation in 5 unknowns. Specifically, the goal is to ascertain $\{\hat{s}, Y, P, N, M\}$ in order to satisfy:

- the indifference condition of the marginal entrant

$$\max_k \int_{\hat{z}(k)} v(z,k)\, f(z \,|\, \hat{s})\, dz = \kappa. \tag{I}$$

- a stationarity condition

$$M \operatorname{Prob}\big(\mathscr{A}\big) = \varphi\, N. \tag{II}$$

- the indifference condition of the marginal potential entrant

$$\int_{\hat{s}} \left(\max_k \int_{\hat{z}(k)} v(z,k)\, f(z|s)\, dz - \kappa \right) f(s)\, ds = \rho. \tag{III}$$

- the economy's prescription of the CES price-index

$$P^{1-\sigma} = F \iint \tilde{p}(z,\tilde{k}(s))^{1-\sigma} f(z,s\,|\,\mathscr{A})\,ds\,dz + V \iint p(z)^{1-\sigma} f(z,s\,|\,\mathscr{A})\,ds\,dz. \quad \text{(IV)}$$

- the labor market clearing condition

$$L = F \iint \tilde{n}(z,\tilde{k}(s)) f(z,s\,|\,\mathscr{A})\,ds\,dz + V \iint n(z) f(z,s\,|\,\mathscr{A})\,ds\,dz + N\phi + E\kappa + M\rho. \quad \text{(V)}$$

In the set of equations above we, moreover, have

$$E = \mathrm{Prob}(\,s \geq \hat{s}\,)\,M$$

while

$$F = (1-\lambda)\,N \quad \text{and} \quad V = \lambda\,N$$

with $\lambda = (1-\theta)/(1-\theta+\varphi\theta)$.

The profit function for a variable-capital firm is then obtained as

$$\pi(z) = \frac{\mu}{(1+\mu)^{\sigma}} \left[\exp(z) \left(\frac{\alpha}{R}\right)^{\alpha} (1-\alpha)^{1-\alpha} \right]^{\sigma-1} P^{\sigma}\,Y - \phi$$

while the profit function for a fixed-capital firm is

$$\tilde{\pi}(z,k) = \left[\frac{1+\mu}{1-\alpha} - 1 \right] \left[\frac{1+\mu}{1-\alpha} \right]^{-\psi\sigma} \left[\frac{1}{\exp(z)\,k^{\alpha}} \right]^{-\psi(\sigma-1)} \left[P^{\sigma}\,Y \right]^{\psi} - Rk - \phi$$

with $\psi^{-1} = 1-\alpha+\alpha\sigma$. It now proves instructive to define $A = P^{\sigma}\,Y$ such that, for an exogenously given R, the value

$$v(z,k) = \frac{\lambda\,\pi(z) + (1-\lambda)\tilde{\pi}(z,k)}{\varphi}$$

is completely pinned down via A. I exploit the recursive structure of the equilibrium equations above, in that I start from an initial guess for A, compute the corresponding $\hat{s}$ via (I), and then evaluate the righthandside of (III). To the extent to which this evaluation differs from ρ, I adjust my guess for A upwards (rhs$<\rho$) or downwards (rhs$>\rho$). I iterate until convergence. With A in hand it is then entirely straightforward to recover M and P via labor market clearing and price index, respectively. Aggregate output Y follows from $A = P^{\sigma}\,Y$.

Appendix B

Proof of Proposition 2: With $s = z + \tau\,\xi$ and ξ independent of z, we have

$$f(z,s) = h(z)\,g\left(\frac{s-z}{\tau}\right)\frac{1}{\tau}.$$

It, thus, follows that

$$f(z\,|\,\hat{s}) = \frac{f(z,\hat{s})}{\int f(z,\hat{s})\,dz} \quad \text{as well as} \quad f(z\,|\,z \geq \hat{z}, s \geq \hat{s}) = \frac{\int_{\hat{s}} f(z,s)\,ds\,\mathbb{1}\{z \geq \hat{z}\}}{\int_{\hat{z}}\int_{\hat{s}} f(z,s)\,ds\,dz}.$$

In order to see how $\hat{z}$ and $\hat{s}$ move with τ, we need to sign B_τ and A_τ. Note that, with

$$\frac{d\,\hat{z}}{d\,\tau} = -\frac{B_\tau}{B_{\hat{z}}} \qquad \text{and} \qquad \frac{d\,\hat{s}}{d\,\tau} = -\frac{A_\tau}{A_{\hat{s}}} - \frac{A_{\hat{z}}}{A_{\hat{s}}}\frac{d\,\hat{z}}{d\,\tau}$$

we have $B_\tau \leq 0 \implies d\,\hat{z}/d\,\tau \leq 0$ while $B_\tau \leq 0$ and $A_\tau \leq 0 \implies d\hat{s}/d\,\tau \leq 0$.

The objective is to sign

$$\frac{\partial B}{\partial \tau} = \int_{\hat{z}}\int_{\hat{s}}\left[\left(\frac{z}{\hat{z}}\right)^{\sigma-1} - 1\right]\frac{\partial}{\partial \tau} f(z,s)\,ds\,dz - \frac{\kappa}{\phi}\int\int_{\hat{s}}\frac{\partial}{\partial \tau} f(z,s)\,ds\,dz.$$

Building on the general convolution structure for $f(z,s)$ and the kernel-symmetry of ξ we have

$$\frac{\partial f(z,s)}{\partial \tau} = \left[h(z)\,g'\left(\frac{s-z}{\tau}\right)\frac{z-s}{\tau^3} - h(z)\,g\left(\frac{s-z}{\tau}\right)\frac{1}{\tau^2}\right] = \ldots$$

$$\left[h(z)\,g\left(\frac{s-z}{\tau}\right)\frac{(s-z)^2}{\tau^4} - h(z)\,g\left(\frac{s-z}{\tau}\right)\frac{1}{\tau^2}\right] = \left[\frac{(s-z)^2}{\tau^2} - 1\right]\frac{f(z,s)}{\tau}.$$

Therefore, at the equilibrium values of $\hat{z}$ and $\hat{s}$, it follows that

$$\frac{\partial B}{\partial \tau} = -\frac{\rho}{\phi}\frac{1}{\tau} + \int_{\hat{z}}\int_{\hat{s}}\left[\exp\big((\sigma-1)(z-\hat{z})\big) - 1\right]\frac{(s-z)^2}{\tau^2}\frac{f(z,s)}{\tau}\,ds\,dz - \ldots$$

$$\frac{\kappa}{\phi}\int\int_{\hat{s}}\frac{(s-z)^2}{\tau^2}\frac{f(z,s)}{\tau}\,ds\,dz.$$

Now, by independence of z and ξ,

$$\frac{\partial B}{\partial \tau} = -\frac{\rho}{\phi}\frac{1}{\tau} + \int_{\hat{z}}\int_{\hat{s}}\left[\exp\big((\sigma-1)(z-\hat{z})\big) - 1\right]\frac{f(z,s)}{\tau}\,ds\,dz\int_{\hat{z}}\int_{\hat{s}}\frac{(s-z)^2}{\tau^2} f(z,s)\,ds\,dz - \ldots$$

$$\frac{\kappa}{\phi}\int\int_{\hat{s}}\frac{(s-z)^2}{\tau^2}\frac{f(z,s)}{\tau}\,ds\,dz.$$

Seeing as $B=0$ at $(\hat{z},\hat{s})$ we can write

$$\frac{\partial B}{\partial\tau}=-\frac{\rho}{\phi}\frac{1}{\tau}+\left[\frac{\rho}{\phi}\frac{1}{\tau}+\frac{\kappa}{\phi}\frac{1}{\tau}\Pr(s\geq\hat{s})\right]\int_{\hat{z}}\int_{\hat{s}}\frac{(s-z)^2}{\tau^2}f(z,s)\,ds\,dz-\frac{\kappa}{\phi}\int\int_{\hat{s}}\frac{(s-z)^2}{\tau^2}\frac{f(z,s)}{\tau}\,ds\,dz.$$

Finally, rearranging terms, we find that

$$\frac{\partial B}{\partial\tau}=\frac{\rho}{\phi\tau}\left[\int_{\hat{z}}\int_{\hat{s}}\frac{(s-z)^2}{\tau^2}f(z,s)\,ds\,dz-1\right]+\ldots$$

$$\frac{\kappa}{\phi\tau}\left[\int_{\hat{z}}\int_{\hat{s}}\frac{(s-z)^2}{\tau^2}f(z,s)\,ds\,dz\,\Pr(s\geq\hat{s})-\int\int_{\hat{s}}\frac{(s-z)^2}{\tau^2}f(z,s)\,ds\,dz\right]\leq 0.$$

■

Proof of Proposition 4: The aggregate markup is constant at

$$1+\mu=\frac{\sigma}{\sigma-1}$$

which is tantamount to saying that the labor share is also constant at

$$\frac{L_p}{PY}=\frac{\sigma-1}{\sigma}.$$

Productive labor in thew expression above is given as $L_p=L-N\phi-E\kappa-M\rho$. Seeing as aggregate profits net of information and entry cost are equal to zero in equilibrium, we have

$$PY=L$$

where we recall that $W=1$ due to our choice of numeraire. Finally, labor market clearing dictates that

$$M\left(\frac{L_p}{M}+\frac{\mathrm{Prob}(\mathscr{A})}{\varphi}\phi+\mathrm{Prob}(s\geq\hat{s})\,\kappa+\rho\right)=L$$

such that equivalently

$$M=\frac{L-L_p}{\mathrm{Prob}(\mathscr{A})\,\phi/\varphi+\mathrm{Prob}(s\geq\hat{s})\,\kappa+\rho}.$$

Overall labor is supplied inelastically at a constant level L. Since, the labor share is constant and aggregate revenue is pinned down via L, it follows that L_p also does not move with τ. Therefore, having established that the numerator in the above expression is constant, it follows that $d\text{Prob}(\mathscr{A})/d\tau \geq 0$ and $d\text{Prob}(s \geq \hat{s})/d\tau \geq 0$ (Proposition 3) imply that $dM/d\tau \leq 0$.

■

Proof of Proposition 5: The planners problem is to maximize aggregate output, i.e. consumption-equivalent welfare

$$Y$$

subject to an aggregation technology

$$M \int_{\hat{z}} \int_{\hat{s}} \Upsilon\left(\frac{y(z)}{Y}\right) F(dz, ds) = 1$$

as well as a labor constraint

$$M\left[\int_{\hat{z}} \int_{\hat{s}} \left(\frac{y(z)}{\exp(z)} + \phi\right) F(dz, ds) + \int_{\hat{z}} \int_{\hat{s}} F(dz, ds) + \rho\right] = L.$$

It is assumed that $\Upsilon \in C^2$, strictly concave, and $\Upsilon(1) = 1$. Optimization takes place via choice of $\{Y, M, \hat{z}, \hat{s}, z \mapsto y(z)\}$. We want to show that the efficient allocation coincides with the market equilibrium allocation if and only is Υ is a power-function, i.e. aggregation takes place in a CES manner such that

$$\Upsilon(x) = x^{\frac{\sigma-1}{\sigma}}.$$

It is well-known (and easy to see) that the equilibrium allocation in a model without information frictions is inefficient iff Υ is not a power-function. The focus of the following proof is, therefore, on demonstrating that – even upon introduction of information frictions - the market equilibrium remains efficient und CES aggregation. With λ and γ respectively denoting the Lagrange multiplier on the technology and labor constraint, the first-order conditions are as follows:

$$Y = \lambda M \int_{\hat{z}} \int_{\hat{s}} \Upsilon'\left(\frac{y(z)}{Y}\right)\left(\frac{y(z)}{Y}\right) F(dz, ds) \tag{Y}$$

$$\lambda \int_{\hat{z}} \int_{\hat{s}} \Upsilon\left(\frac{y(z)}{Y}\right) F(dz, ds) = \gamma\left[\int_{\hat{z}} \int_{\hat{s}} \left(\frac{y(z)}{\exp(z)} + \phi\right) F(dz, ds) + \int_{\hat{z}} \int_{\hat{s}} F(dz, ds) + \rho\right] \tag{M}$$

$$\lambda \Upsilon\left(\frac{y(\hat{z})}{Y}\right) = \gamma\left[\frac{y(\hat{z})}{\exp(\hat{z})} + \phi\right] \tag{$\hat{z}$}$$

$$\lambda \int_{\hat{z}} \Upsilon\left(\frac{y(z)}{Y}\right) f(z, \hat{s}) dz = \gamma\left[\int_{\hat{z}} \left(\frac{y(z)}{\exp(z)} + \phi\right) f(z, \hat{s}) dz + \kappa \int f(z, \hat{s})\, dz\right] \tag{$\hat{s}$}$$

$$\lambda\Upsilon'\left(\frac{y(z)}{Y}\right)\frac{1}{Y}=\gamma\exp(-z) \qquad (z\mapsto y)$$

Dividing the first-order condition for $z\mapsto y(z)$ by the first-order condition for $\hat{z}$ and evaluating at $\hat{z}$, we obtain

$$\frac{\Upsilon'\left(\frac{y(\hat{z})}{Y}\right)\frac{y(\hat{z})}{Y}}{\Upsilon\left(\frac{y(\hat{z})}{Y}\right)}=\frac{y(\hat{z})\exp(-\hat{z})}{y(\hat{z})\exp(-\hat{z})+\phi} \qquad \rightsquigarrow_{\text{CES}} \qquad y(\hat{z})=(\sigma-1)\phi\exp(\hat{z}).$$

From this we can recover a general allocation function $z\mapsto y(z)$ since

$$\frac{\Upsilon'\left(\frac{y(z)}{Y}\right)}{\Upsilon'\left(\frac{y(\hat{z})}{Y}\right)}=\frac{\exp(\hat{z})}{\exp(z)} \qquad \rightsquigarrow_{\text{CES}} \qquad y(z)=(\sigma-1)\,\phi\,\exp\big((\sigma-1)(z-\hat{z})\big)\,\exp(z).$$

From the labor resource constraint and the first-order condition w.r.t. M, we have

$$\lambda\int_{\hat{z}}\int_{\hat{s}}\Upsilon\left(\frac{y(z)}{Y}\right)F(dz,ds)=\gamma\frac{L}{M} \quad\Longleftrightarrow\quad \frac{M}{L}\int_{\hat{z}}\int_{\hat{s}}\Upsilon\left(\frac{y(z)}{Y}\right)F(dz,ds)=\frac{\Upsilon\left(\frac{y(\hat{z})}{Y}\right)\exp(\hat{z})}{y(\hat{z})+\phi\exp(\hat{z})}$$

where the latter follows from the first-order condition w.r.t. $\hat{z}$. With a power-function specification of Υ we can, therefore, see that

$$\frac{M}{L}\int_{\hat{z}}\int_{\hat{s}}\exp\big((\sigma-1)(z-\hat{z})\big)F(dz,ds)=\frac{1}{\sigma\phi}.$$

It is also immediate (from the labor resource constraint) that

$$\frac{M}{L}\left[(\sigma-1)\phi\int_{\hat{z}}\int_{\hat{s}}\exp\big((\sigma-1)(z-\hat{z})\big)F(dz,ds)+\kappa\int_{\hat{s}}f(s)ds+\rho+\phi\int_{\hat{z}}\int_{\hat{s}}f(z,s)ds\,dz\right]=1.$$

Under CES aggregation, the prescription of the social planner, therefore, entails the first selection equation obtained for the market equilibrium. That is,

$$\int_{\hat{z}}\int_{\hat{s}}\left[\mathrm{e}^{(\sigma-1)(z-\hat{z})}-1\right]F(dz,ds)=\frac{\kappa}{\phi}\iint_{\hat{s}}F(dz,ds)+\frac{\rho}{\phi}$$

Finally, the first-order condition w.r.t $\hat{s}$ dictates that

$$\mathbb{E}\left[\Upsilon\left(\frac{y(z)}{Y}\right)\mathbb{1}\{z\geq\hat{z}\}\,\middle|\,\hat{s}\right]=\frac{\gamma}{\lambda}\mathbb{E}\left[\left(\frac{y(z)}{\exp(z)}+\phi\right)\mathbb{1}\{z\geq\hat{z}\}\,\middle|\,\hat{s}\right]+\frac{\gamma}{\lambda}\kappa$$

where

$$\frac{\gamma}{\lambda} = \Upsilon'\left(\frac{y(\hat{z})}{Y}\right)\frac{\exp(\hat{z})}{Y}$$

where, under CES aggregation, we can substitute $\exp(\hat{z}) = y(\hat{z})/(\sigma-1)/\phi$. It, thus, follows that

$$\int_{\hat{z}} \left[\exp\big((\sigma-1)(z-\hat{z})\big)-1\right] f(z|\hat{s})\,dz = \frac{\kappa}{\phi}$$

which is the second selection equation in the decentralized allocation.

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